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TRANSIENTS IN CYLINDRICAL ANTENNAE

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SUMMARY

The transient response of the radiation field of a driven cylindrical antenna is investigated for the particular case of a step-function excitation. The theoretical analysis makes use of Fourier's theorem to express the response as an integral over the response to all individual frequency components. The response as a function of time shows damped oscillations with a frequency determined by the first resonance frequency of the antenna. The response of the same antenna used as a receiver in a transient plane wave field is shown to be related to the radiation response by a simple integration process. By proper loading of the dipole, transient times of the order of the time needed for a wave to travel along the dipole axis can be obtained. An experimental investigation is described in which the reception of a transient field due to a shock-excited distant transmitter is observed.

LIST OF SYMBOLS

a = Radius of cylindrical antenna.
c = Velocity of light.
h = Half-length of cylindrical antenna.
h_e = Effective half-length of cylindrical antenna.
r, θ, ψ = Co-ordinates of spherical co-ordinate system.
r_1, r_2 = Distances from centre of antenna to reference points.
x, y, z = Co-ordinates of rectangular co-ordinate system.
E = Electric field strength.
E_0 = Amplitude of incident electric field.
$E_y^{rad}, E_\theta^{rad}$ = Components of radiated electric field.
$F\left(\theta, \beta h, \frac{a}{h}\right)$ = Field factor of antenna.
I = Current at centre point of driven antenna.
I_L = Current at centre point of receiving antenna.
R_c = Characteristic resistance of connecting cable.
R_g = Internal generator resistance.
R_L = Load resistance of receiving antenna.
V = E.M.F. of driving generator.
V_0 = Amplitude of generator e.m.f.
V_{oc} = E.M.F. at centre point of receiving antenna.
V_L = Voltage across load of receiving antenna.
T = Transient time.
Z_g = Internal impedance of generator.
Z_{in} = Input impedance of cylindrical antenna.
Z_L = Load impedance of receiving antenna.
$\alpha = c\tau/h$ = Relative time scale.
$\beta = 2\pi/\lambda$ = Phase-change coefficient.
ϵ = Permittivity.
ψ' = Expansion parameter.
λ = Wavelength.
μ = Permeability.
ω = Angular frequency.
$r = 2 \log 2h/a$ = Logarithmic measure for relative antenna thickness.
$\left.\begin{array}{l}\tau_1 = t - r_1/c \\ \tau_2 = t - r_2/c\end{array}\right\}$ = Shifted-time-scale co-ordinates.

(1) INTRODUCTION

For a harmonic time dependence of the excitation the radiation properties and circuit properties of linear antennae are found by solving the time-independent Maxwell equations with the appropriate boundary conditions at the driving point or load point and on the surface of the antenna.[1,2] Important applications of antennae, however, involve the situation where, at a certain time t_0, one or more of the boundary conditions (usually the exciting voltage or field) are rapidly changed. Whenever such a change is made, transient oscillations occur in the antenna due to the finite bandwidth of the system. These transient currents provide the continuous transition from the stationary distribution of charges and currents in the state I for $t < t_0$ to the final distribution of charges and currents in the state II long after the change in conditions is effected.

Practical cases involving transients in antennae include sudden changes in the driving voltage, such as in the transmission of short pulses, and rapid non-periodic changes of the exciting field at a receiving antenna such as are encountered in lightning or similar electromagnetic disturbances. Another application where use is made of transient oscillations in antennae is the generation of millimetre waves by spark-excited small particles in the so-called 'mass generator'.

(2) RECIPROCITY RELATION BETWEEN TRANSMITTING AND RECEIVING ANTENNAE

In the steady state the radiation field of a centre-driven electric dipole of length $2h$ and wire radius a [Fig. 1(a)] is given by[3]

$$E_\theta^{rad} = \frac{jI\sqrt{\frac{\mu}{\epsilon}}}{2\pi r_1} F\left(\theta, \beta h, \frac{a}{h}\right) \varepsilon^{-j\beta r_1} \quad . \quad . \quad . \quad (1)$$

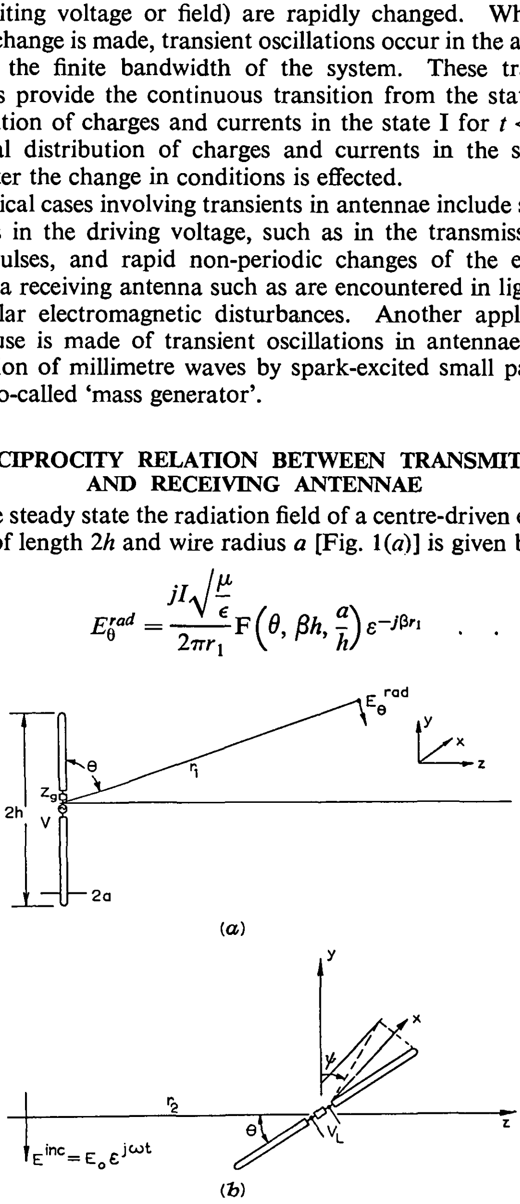

Fig. 1.—Geometrical configuration of the antennae.

(a) Driven antenna.
(b) Receiving antenna.

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In terms of the e.m.f. of the driving generator $V = V_0\varepsilon^{j\omega t}$ and with $\sqrt{(\mu/\epsilon)} = 120\pi$ ohms the radiation field becomes

$$E_0^{rad} = \frac{jV_0 60}{Z_{in} + Z_g}\mathrm{F}\left(\theta, \beta h, \frac{a}{h}\right)\frac{\varepsilon^{j\omega[t-(r_1/c)]}}{r_1} \quad . \quad . \quad (2)$$

If the same antenna, used as a receiver, is placed at the origin of the co-ordinate system, inclined against the direction of the electric vector of an incident linearly polarized plane wave by angles θ and ψ [Fig. 1(*b*)] the induced open-circuit voltage V_{oc} at the terminals of the antenna is, by definition of the 'effective half-length' h_e (see Reference 3, p. 467),

$$V_{oc} = -\,2h_e\left(\theta, \beta h, \frac{a}{h}\right)E_0\varepsilon^{j\omega[t-(r_2/c)]}\cos\psi \quad . \quad . \quad (3)$$

The term $\varepsilon^{-j\omega r_2/c}$ gives the phase measured against a point in space at a distance r_2 from the centre of the dipole. By the application of Thévenin's theorem the induced current through the load and the voltage V_L across the load impedance becomes

$$I_L = \frac{V_L}{Z_L} = \frac{V_{oc}}{Z_{in} + Z_L} = \frac{-2h_e\left(\theta, \beta h, \frac{a}{h}\right)}{Z_{in} + Z_L}E_0\varepsilon^{j\omega[t-(r_2/c)]}\cos\psi \qquad (4)$$

Applying the Rayleigh–Carson reciprocity theorem it can be proved (see Reference 3, p. 570) that it always holds that

$$\mathrm{F}\left(\theta, \beta h, \frac{a}{h}\right) = \beta h_e\left(\theta, \beta h, \frac{a}{h}\right) \quad . \quad . \quad . \quad (5)$$

i.e. the receiving pattern and the transmitting pattern are the same for a given antenna.

Upon differentiating eqn. (4) with respect to time and using eqn. (5) it is seen that, in the special case when $Z_L = Z_g$,

$$\frac{\partial I_L}{\partial t} = -\,2jE_0c\frac{\beta h_e\left(\theta, \beta h, \frac{a}{h}\right)}{Z_{in} + Z_g}\varepsilon^{j\omega\tau_2}\cos\psi$$

$$= \frac{-\,2E_0cr_1\cos\psi}{V_0 60}\varepsilon^{j\omega(\tau_2-\tau_1)}E_\theta^{rad} \qquad (6)$$

Thus the response of a receiving antenna differentiated with respect to time is proportional to the time dependence of the radiated electric field at a fixed point in space given by θ and ψ. The constant of proportionality depends neither on the particular antenna size nor on the frequency. If the excitation is of non-sinusoidal time dependence, the signal may be represented by its Fourier spectrum, and since eqn. (6) holds for any single component, it also holds for the composite signal. Hence, with a knowledge of the induced current I_L on a receiving antenna due to an incident wave of arbitrary time dependence, the radiated field of an identical transmitting antenna excited by a voltage of the same time dependence can be computed directly by differentiation. If the radiated transient field of a driven antenna is known, the transient current in a receiving antenna can be computed by integration.

(3) TRANSIENT RESPONSE

As a particular case, the broadside ($\theta = 90°$, $\psi = 0°$) radiation from a dipole excited by a voltage that is a step function in its time dependence is considered. The Fourier representation of the voltage step is

$$V(t) = \frac{V_0}{2} + \frac{V_0}{\pi}\int_0^\infty \frac{\sin\omega t}{\omega}d\omega \quad . \quad . \quad . \quad . \quad (7)$$

where the sudden rise in the signal occurs at $t = 0$ at the dipole centre. The field due to transient currents is defined by

$$E(t) = E_{trans}(t) + E_{\mathrm{II}}(t) \quad . \quad . \quad . \quad . \quad (8)$$

where $E(t)$ is the actual field under investigation at any time $t > 0$ and $E_{\mathrm{II}}(t)$ is the final field in the new steady state II after the transient part has died out. Since it is known that the radiation field is zero for static excitation, $E_{\mathrm{II}}(t)$ vanishes and the total field is due to transient currents only.

For a driving voltage of time dependence sin ωt, the response is given by the imaginary part of eqn. (2). Thus, owing to the linearity of all components, the application of eqn. (7) to eqn. (2) yields for the broadside radiation field of a dipole excited at the time $t = 0$ by a voltage step

$$E_\theta^{rad}(t) = E_y^{rad}(t) = \frac{60V_0}{\pi r_1}\mathscr{I}\left[\int_0^\infty \frac{j\beta h_e\left(90°, \beta h, \frac{a}{h}\right)\varepsilon^{j\omega\tau_1}}{(Z_{in} + Z_g)\omega}d\omega\right] \quad (9)$$

Let the variable of integration be changed from ω to $\beta h = \omega h/c$.

$$E_\theta^{rad}(t) = \frac{V_0 60}{r_1\pi}\left[\int_0^\infty A(\beta h)\cos\beta h\left(\frac{c}{h}\tau_1\right)d\beta h\right.$$

$$\left.+ \int_0^\infty B(\beta h)\sin\beta h\left(\frac{c}{h}\tau_1\right)d\beta h\right] \quad (10)$$

$$\frac{A(\beta h)}{[B(\beta h)]} = \frac{\mathscr{I}}{[\mathscr{R}]}\left[\frac{j\beta h_e\left(90°, \beta h, \frac{a}{h}\right)}{(Z_{in} + Z_g)\beta h}\right]$$

The retardation of the total response at the point of observation is contained in $\tau_1 = t - \frac{r_1}{c}$.

Since the spectrum of the applied step-function signal extends to infinite frequencies, the integrations in eqn. (10) have to be extended to infinity. However, numerical values of the input impedance and the complex effective length from the second-order expansion (Reference 3, pp. 154 and 488) or the variational solution[2,4] are known only up to $\beta h \simeq 5$, thus just including the second resonance frequency of a centre-driven dipole of length $2h$. For a particular ratio of thickness to length of the antenna $a/h = 1/904$ [$\Omega = 2\log(2h/a) = 15$] and zero generator impedance, the quantities $A(\beta h)$ and $B(\beta h)$ are plotted as a function of βh in Fig. 2. It is seen that the values decrease sufficiently with increasing βh (increasing frequency for fixed antenna length) so that the integration may be discontinued at $\beta h \simeq 5$ without introducing a large error.

In order to obtain some quantitative results for the transient response due to a step-function input signal, the integrations (10) have been carried out graphically by means of a planimeter for different values of τ_1. The computed response of the far-zone field, assuming a generator with negligible internal impedance ($Z_g = 0$), is plotted in Fig. 3. The abscissa is the dimensionless quantity

$$\alpha = \frac{c\tau_1}{h} = \frac{c}{h}\left(t - \frac{r_1}{c}\right)$$

The quantity α measures the time in terms of the time required by an electromagnetic wave to travel a distance h, i.e. from the dipole centre to the tips of the dipole. It is seen that the main part of the signal starts at an instant $\alpha = 0$, and thus with a retardation r_1/c. For positive values of α the response jumps rather rapidly from positive to negative values, resembling a pulse-shaped curve. It can be seen from Fig. 2 that the main

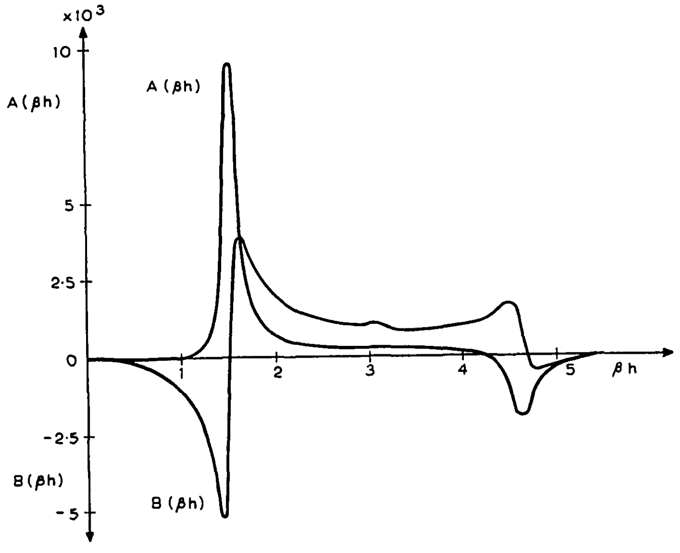

Fig. 2.—$A(\beta h)$ and $B(\beta h)$ for $\Omega = 15$.
Numerical values from King–Middleton expansion, $Z_g = 0$.

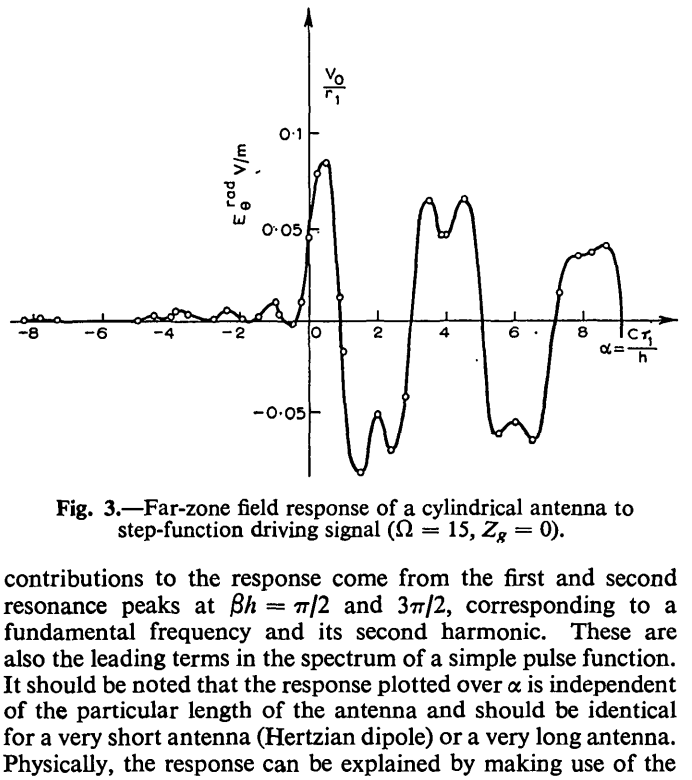

Fig. 3.—Far-zone field response of a cylindrical antenna to step-function driving signal ($\Omega = 15$, $Z_g = 0$).

contributions to the response come from the first and second resonance peaks at $\beta h = \pi/2$ and $3\pi/2$, corresponding to a fundamental frequency and its second harmonic. These are also the leading terms in the spectrum of a simple pulse function. It should be noted that the response plotted over α is independent of the particular length of the antenna and should be identical for a very short antenna (Hertzian dipole) or a very long antenna. Physically, the response can be explained by making use of the close analogy between the electric dipole antenna and an open-ended 2-wire transmission line. In the first moment after the switching operation, $0 < \alpha < 1$, positive charges travel in one arm of the dipole towards the antenna tip and negative charges in the other arm (Fig. 4). The associated currents set up a strong electric field in the far zone. At $\alpha = 1$ a reflection of the electromagnetic field associated with the moving charges takes place at the antenna tips, which, by analogy with the open-ended transmission line have a positive reflection factor. Hence, for $1 < \alpha < 2$, charges of the same sign as before travel in the opposite sense and the associated currents are reversed. For $\alpha = 2$ a reflection at the centre of the dipole takes place, where, by analogy with the transmission line, the lossless generator gives a reflection factor of -1. Hence, for $2 < \alpha < 3$, discharge waves travel from the dipole centre outward, and thus the field remains negative until, at $\alpha = 3$, the direction of the discharge waves is reversed and the whole process starts repeating itself at $\alpha = 4$. The radiation field will have the largest amplitude when the moving discontinuity of charges is close to the antenna tips. It will be weaker if the discontinuity is close to the gap. This explains the marked dip in the response for $\alpha = 2$, and in the succeeding peaks. Owing to the radiation of energy the transient signal at any point of observation dies out after a number of successive maxima and minima.

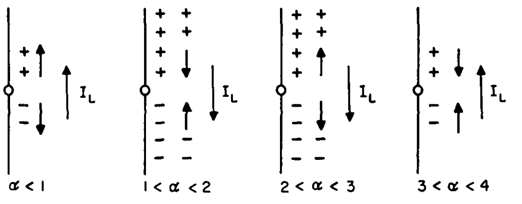

Fig. 4.—Total charges and currents in a step-function-excited transmitting antenna.

Owing to the finite velocity of electromagnetic waves no signal should occur for $\alpha < 0$. Strictly, in eqn. (10) the condition

$$\int_0^\infty A(\beta h)\cos\beta h\left(\frac{c}{h}\tau_1\right)d\beta h = \int_0^\infty B(\beta h)\sin\beta h\left(\frac{c}{h}|\tau_1|\right)d\beta h \quad (11)$$

should hold. Since the numerical values of input impedance and effective length of the antenna are derived from non-physical solutions (second-order expansion or variational solution), condition (11) is only approximately fulfilled, giving rise to a small ($\sim$10% of the main peak) signal for negative times. This error also includes the effect of the discontinued integration.

From the simplified explanation of the transient process in terms of charges and currents on a transmission line it is evident that the decay of the transient signal can be greatly influenced by the internal impedance of the generator. Investigations have been made for various generator resistances and antenna thicknesses. It is found that the transient time T in terms of a decay of $1/\varepsilon$ of the power decreases first with increasing generator resistance, reaches a minimum at around $R_g = 600$ ohms for $\Omega = 15$ and slowly rises again thereafter. The computed transient response of the far-zone field due to a step-function excitation for the critical value of $R_g = 600$ ohms is plotted in Fig. 5. Apparently the electromagnetic wave, reflected at the

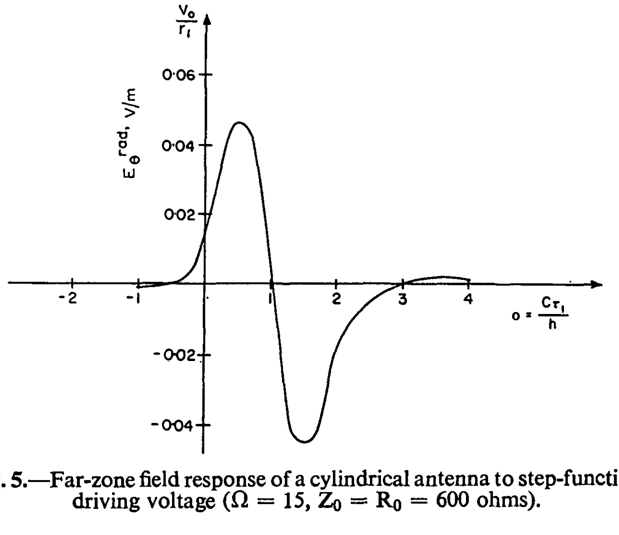

Fig. 5.—Far-zone field response of a cylindrical antenna to step-function driving voltage ($\Omega = 15$, $Z_0 = R_0 = 600$ ohms).

dipole tips and travelling back to the dipole centre is reflection-free absorbed in the generator resistance. The total transient field lasts just the time needed for a wave to travel from dipole centre to the tip and back, in complete analogy to the transient behaviour of an open-ended section of transmission line excited by a generator with an impedance equal to the characteristic resistance of the line. In the zeroth-order antenna theory (Reference 3, p. 144) the input impedance of cylindrical antennae is given by the impedance of such an open-ended section of transmission line with a characteristic resistance

$$R_c = \sqrt{\frac{\mu}{\epsilon}} \Big/ 2\pi\psi' = 60\psi' = 60(\Omega - 2)$$

Hence, for $\Omega = 15$, the value of R_c is 780 ohms. This is the same order of magnitude as deduced from the investigation of the transient process. This simple relation also shows correctly that, for thicker antennae (smaller values of Ω), the critical resistance is smaller. Recalling the relation between transient time T and the bandwidth of any resonant system Δf (measured at half-energy points on the resonance curve),

$$T \sim \frac{1}{\Delta f} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

it is apparent that a driving source with this internal resistance also maximizes the useful bandwidth of the antenna.*

Using the reciprocity relation (6), the response of a centre-loaded receiving antenna to a transient plane-wave field of step-function time dependence (broadside incidence) is easily found by integrating the radiation field response in time. The received transient signal at the centre terminals for $Z_L = 0$ and for a load resistance $R_L = 600$ ohms are plotted in Figs. 6 and 7, respectively. For the unloaded antenna the response again has the shape of a weakly-damped oscillation with a fundamental frequency determined by the first resonance of the antenna. The decay of the transient signal is entirely due to re-radiation. A physical interpretation of the transient process in terms of charges and currents can be given as follows. Initially, before the arrival of the signal, the antenna is uncharged so that positive and negative charges are equally distributed. In the final steady state for t sufficiently long after the arrival of the step signal, the antenna is in a static field. Thus charges will be induced on the surface of the dipole, concentrated close to the antenna ends and with opposite signs at the lower and the upper end. Analogous to eqn. (8) the distribution of total charges is described for any time $\tau > 0$ by a superposition of the steady-state distribution for t very large and the transient part. Since the total charge is zero for $\tau = 0$, the distribution of charges that give rise to the transient currents in the antenna initially has to be similar to that in the final steady state, but with charges of the opposite signs (see Fig. 8). The relaxation of these charges for $\tau > 0$ corresponds closely to the discharge of a charged section of transmission line.[5]

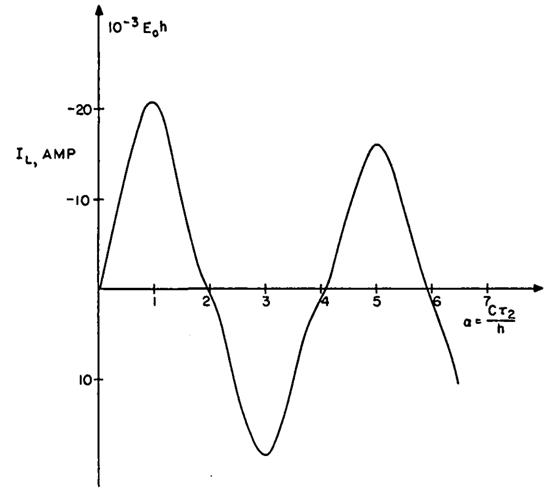

Fig. 6.—Transient response of the current at the dipole centre due to step-function rise in field strength.
Broadside incidence, $Z_L = 0$, $\Omega = 15$, obtained from Fig. 3.

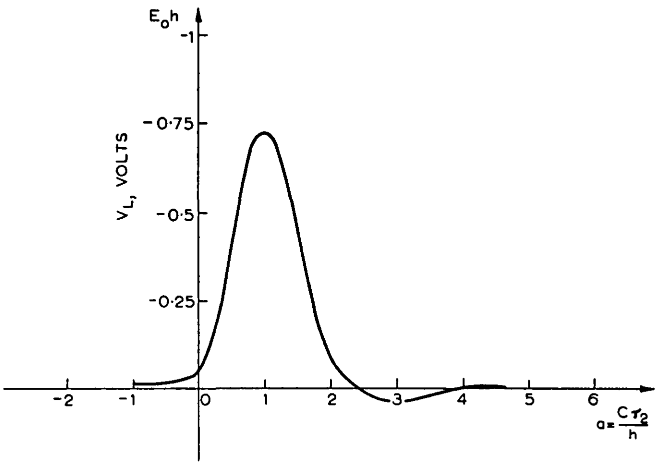

Fig. 7.—Transient response of voltage across load resistor (R_L = 600 ohms) due to step-function rise in field strength.
Broadside incidence, $\Omega = 15$.

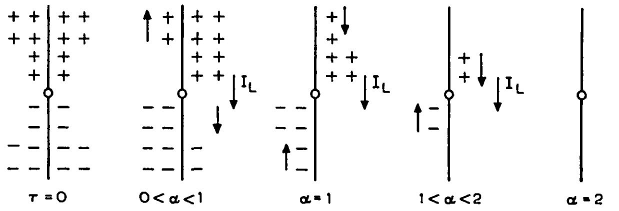

Fig. 8.—Transient currents at the dipole centre, and charges in a loaded receiving antenna.

Particularly interesting is the case when the load impedance corresponds to the characteristic impedance of the equivalent transmission line (Fig. 7). The transient response then shows a single peak with a total transient time exactly twice that needed for a wave to travel from dipole centre to the dipole tips. The instantaneous distribution of charges connected with the transient process* and the current through the dipole terminals are indicated in Fig. 8. The current in the peak at $\alpha = 1$ is very nearly one-half the peak current for the unloaded antenna.

(4) EXPERIMENTS

Since no ideally broad-band field probes or transmitters are available, the radiated electric field of a shock-excited dipole antenna and the received voltage of an incident wave that is a step function cannot easily be investigated experimentally. However, the combined reception of a transient signal due to a transient-excited transmitter forms a complete system that can be realized experimentally and may serve as a check on the accuracy of the computations.

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* For an antenna loaded at the tips (travelling-wave antenna) the transient time may be less.

* In order to obtain the total charges the distribution in the steady state II has to be superimposed.

For one particular case, $R_L = 120$ ohms and $Z_g = 0$ (impedanceless generator), the transient voltage across the load of the receiving antenna due to a step-function excitation at $t = 0$ of the distant transmitting antenna has been computed by combining eqns. (2) and (4) and subsequent graphical integration. Again $\Omega = 15$ is chosen and both dipoles are equal in length and parallel to each other. The total response, as calculated from numerical values of the variational solution, is plotted in Fig. 9. The electric field radiated from the distant transmitter is the one represented in Fig. 3. It arrives at the time $\tau_1 = (t - r_1/c) = 0$ at the loaded receiving antenna and induces the transient voltage V_L, which shows almost a sinusoidal character. The total attenuation of the transient signal is due to the overall decrease in the incident field, to re-radiation, and to losses in the load resistance. Since the antennae are assumed to be in the far zone with respect to each other, no mutual coupling is included. The response shown in Fig. 9 is independent of the antenna length, since an ideal step-function input signal is assumed. Only the amplitude of the transient response is linearly proportional to the antenna length.

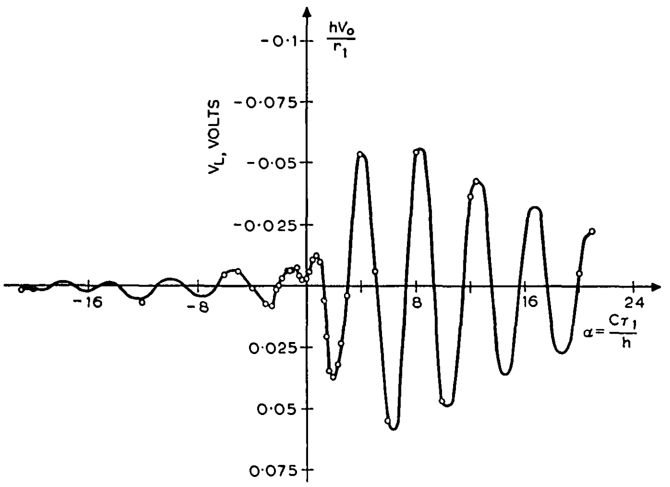

Fig. 9.—Transient voltage across load resistance of receiving antenna in far-zone field of identical antenna excited by step-function voltage.

$\Omega = 15$, $R_L = 120$ ohms, $Z_g = 0$.

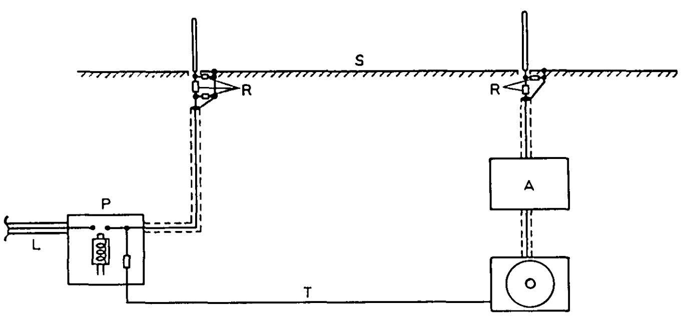

Fig. 10.—Experimental arrangement for measuring transient responses from cylindrical antennae.

S = Ground screen.
R = Resistive networks.
P = Fast-rise pulse generator.
L = Pulse-forming line.
A = Wide-band chain amplifiers.
O = Oscillograph.
T = Trigger line.

For the experiment, the arrangement shown schematically in Fig. 10 was used. The main parts consist of a large 24 × 48 ft metallic earth screen on top of the laboratory building, and two metallic rods of variable length as cylindrical antennae. The transmitting antenna is fed by a Spencer Kennedy fast-rise pulse generator. The rectangular d.c. pulse output is generated by connecting a previously charged coaxial line through a mercury switch to a load resistance equal to the characteristic resistance of the charged line. In the experiment a very long pulse-forming coaxial line has been used so that the negative step occurs at a time when the primary transient process due to the sudden rise in the signal has long decayed. The voltage step with a rise time of less than 1 millimicrosec is about 150 volts, and the repetition rate is 130 pulses/sec. It is fed through a long coaxial cable of matched characteristic resistance ($R_c = 50$ ohms) to the driven antenna above the earth screen. Between antenna and coaxial line a resistive network is used to match the coaxial line* and to represent different generator resistances for the antenna. Account is taken of the fact that antenna and resistance have an image in the earth screen, so that load and generator resistances are actually twice the value represented by the network. Since the voltage step has a finite rise time, care has to be taken to make this fact unimportant for a comparison with the theoretical response. The criterion here is that the rise time should be short compared with the time a wave needs to travel along the antenna. With an antenna length of 121 cm (travel time from centre to top of about 4 millimicrosec) this condition is reasonably well fulfilled.

The receiving dipole is located at a distance of about 4 m from the driven antenna with its axis parallel to it. The received signal is fed through a resistive network to represent different loads into a long coaxial cable with $R_c = 120$ ohms. The signal is amplified in several wide-band chain amplifiers with a cut-off frequency of 300 Mc/s, and finally fed directly to the vertical deflection plates of a high-speed oscillograph. The sweep of the instrument has a minimum deflection time of 10 millimicrosec/cm and is externally triggered by a small fraction of the pulse output from the pulse generator.

To test the total equipment the two coaxial lines leading to and from the antennae were connected directly, and the pulse obtained was photographed. The rise time was about 1 millimicrosec, with some small oscillations on the top, which were found to be due to transients in the resonant circuit formed by the capacitance of the deflection plates and the lead inductances of the oscillograph. Several images of this pulse due to reflections could be seen on the oscillograph; all of these images, however, arrive at a much later time.

In the first experiments all conditions assumed for the theoretical curve (Fig. 9) were realized. The impedance looking from driven antenna into the generator was adjusted to 10 ohms, which is negligible compared with the real part of Z_{in} at almost any frequency. The load resistance at the receiver side was 120 ohms. With a brass rod of radius $a = \frac{1}{16}$ in (1·52 mm), $\Omega = 14{\cdot}7 - 15$. The photographed response with different sweep times is shown in Fig. 11. It is apparent that it shows the same overall character as the theoretically computed response in Fig. 9. The time for one complete transient oscillation can best be read from the picture with a sweep speed of 20 millimicrosec to be about 16·5 millimicrosec, which compares with the theoretical value of 16·7 millimicrosec. Also, the typical shape of the build-up of the transient oscillations is clearly indicated. The first peak (owing to phase reversal it appears in the positive direction) is much smaller than the ensuing peaks. An evaluation of T for a decrease to $\varepsilon^{-1/2}$ of the largest amplitude gives

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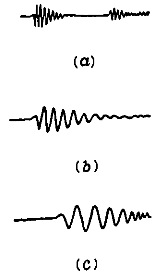
* Actually, this would not be necessary since a sufficiently long cable is used so that eventual reflections occur after the primary transient has died out.



(a)

(b)

(c)

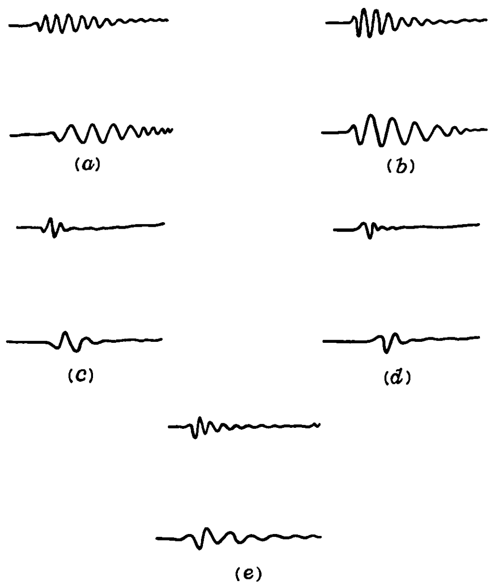
Fig. 11.—Transient voltage across load of receiving antenna in far-zone field of an identical antenna excited by step-function voltage.

$\Omega = 14 \cdot 7$.
$R_L = 120$ ohms.
$R_g = 10$ ohms.
$h = 121$ cm.

(a) 50 millimicrosec/cm sweep.
(b) 20 millimicrosec/cm sweep.
(c) 10 millimicrosec/cm sweep.

70·5 and 68 millimicrosec from theory and experiment, respectively. The amplitude of the received signal has been measured and agrees to within 1 dB with the theoretical value.*

In order to investigate whether the antenna length is already sufficient, in view of the finite rise time of the pulse, the response has been measured for different antenna lengths. It was found that, with decreased antenna length, differences between theory and experiment occur only if antenna length and the product of rise time of the pulse and phase velocities become comparable.



(a) (b)

(c) (d)

(e)

Fig. 12.—Transient voltage across load of receiving antenna in the far-zone field of an identical antenna excited by step-function voltage.

$\Omega = 14 \cdot 7$.
$h = 121$ cm.

Sweep rate 20 and 10 millimicrosec/cm.

(a) $R_L = R_g = 30$ ohms.
(b) $R_L = R_g = 120$ ohms.
(c) $R_L = R_g = 400$ ohms.
(d) $R_L = R_g = 800$ ohms.
(e) $R_L = R_g = 3\,000$ ohms.

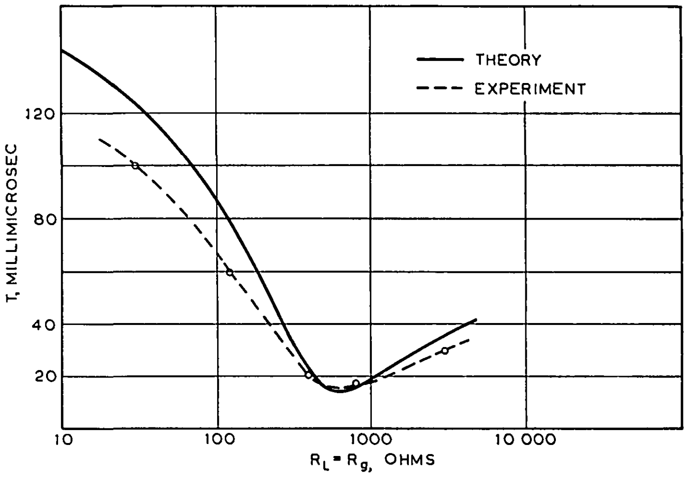


Fig. 13.—Transient time of voltage across load induced by distant transmitter excited with voltage step.

$\Omega = 15$.
$h = 121$ cm.

In the next series of measurements the dependence of T on the generator and load resistances is investigated for antennae with $h = 121$ cm and $\Omega \simeq 15$. The resistive networks are adjusted in each case to give the same resistances, presented from the antennae to the networks ($R_L = R_g$). The photographed responses are shown in Fig. 12. The period of transient oscillations is in all cases about equal. The total transient time is seen to be critically dependent on the load impedances. The transient times measured from these photographs are plotted in Fig. 13, and, for comparison, the transient times obtained theoretically are included. Because of the inaccuracy in reading the $\varepsilon^{-1/2}$ amplitude decrease from the photographs, the agreement cannot be expected to be more than qualitative. However, the expected minimum of the transient time is clearly indicated in the measured results, and the position of the minimum at about 600 ohms agrees well with the approximate theory.

(5) CONCLUSIONS

From a comparison of the experimental results and the theoretically computed curve for the response in the combined transmission-reception case the conclusion can be drawn that the theory adequately describes the transient process in a cylindrical antenna. Because of the close agreement between the values obtained from the variational method and those from the second-order King–Middleton expansion, this applies to both theoretical formulations.

It is found that, in order to minimize the transient time of cylindrical antennae, they should be loaded with a resistance of about 600 ohms for practical antennae ($\Omega = 15$). This will be important in all cases where a correct transmission of wide-band signals is desired. It is, however, at the cost of sensitivity, particularly in the case of a driven antenna.

If interest lies rather in the generation of long sustained transient oscillations, impedanceless generators should be used, such as would be approximated by a spark between the narrow terminals in a driven antenna. Thin wires should also be used.

For the analysis of general transient processes it could be shown that only the receiving or transmitting case has to be investigated, since each case is connected to the other one by a differentiation or integration process.

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* On the 10 millimicrosec/cm sweep position, the sweep was found to be not exactly linear, showing the correct deflection rate only very close to the starting-point, as can be seen from the photographed response. In the picture taken with 50 millimicrosec/cm deflection, the transient due to the negative step of the rectangular pulse enters the picture. This step is not quite so sharp and gives a somewhat smaller response signal.

(6) ACKNOWLEDGMENT

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(7) REFERENCES

(1) King, R. W. P., and Middleton, D.: 'The Cylindrical Antennas; Current and Impedance', *Quarterly of Applied Mathematics*, 1946, **3**, p. 302.

(2) Storer, J. E.: 'Variational Solution to the Problem of the Symmetrical Cylindrical Antenna', Technical Report 101, Cruft Laboratory, Harvard University, February, 1950.

(3) King, R. W. P.: 'The Theory of Linear Antennas' (Harvard University Press, 1956), p. 529.

(4) Schmitt, H. J.: 'Transients in Cylindrical Antennas', Technical Report 296, Cruft Laboratory, Harvard University, January, 1959.

(5) Wagner, K. W.: 'Elektromagnetische Ausgleichsvorgaenge in Freileitungen und Kabeln' (B. G. Teubner Verläg, 1908).